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$\Lambda_b \to \Sigma_c^{(*)} \pi l \bar{\nu}$ in the heavy quark, chiral, and large N_c limits

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Abstract

The decay of $\Lambda_b \to \Sigma_c^{(*)} \pi l \bar{\nu}$ processes are calculated in the framework of heavy baryon chiral perturbation theory with help of the $1/N_c$ expansion. The decay rate distributions over $v \cdot v'$ where v and v' are the 4-vector velocities of Λ_b and $\Sigma_c^{(*)}$ respectively, and over the pion and lepton energies are given. The decay branching ratios amount to about 1.21(0.971)% for $\Lambda_b \to \Sigma_c \pi l \bar{\nu}$ and 0.798(0.640)% for $\Lambda_b \to \Sigma_c^* \pi l \bar{\nu}$ by using the baryonic Isgur-Wise function predicted by QCD sum rule (large N_c QCD).

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I. INTRODUCTION

Heavy baryons provide a testing ground to the standard model (SM). Experiments are accumulating more and more data on heavy baryons [1]. For the theoretical calculation, the task is to apply model-independent methods of the nonperturbative QCD. For the heavy hadrons containing a single heavy quark, the heavy quark effective theory (HQET) [2] can be applied successfully. After combining the chiral symmetry, the low energy effective theory of HQET, the heavy hadron chiral Lagrangian (HHCL) [3] extends the application area further. Recently, the $1/N_c$ expansion approach [4] incorporates in more understanding of QCD to the baryon sector.

The heavy baryon decays $\Lambda_b \to \Sigma_c^{(*)}\pi l\bar{\nu}$ [5] are interesting from the point of view of the above mentioned approaches. In the heavy quark limit, the most interesting weak transitions $\Lambda_b \to \Lambda_c$ and $\Sigma_b^{(*)} \to \Sigma_c^{(*)}$ are simplified greatly. The numbers of the independent form factors are reduced to one and two, respectively in these two transitions. The decays $\Lambda_b \to \Sigma_c^{(*)} l\bar{\nu}$ [6] however, are predicted to be both $1/m_Q$ and $1/N_c$ suppressed. They provide a test to the HQET. The decays $\Lambda_b \to \Sigma_c^{(*)}\pi l\bar{\nu}$, on the other hand, are neither $1/m_Q$ nor $1/N_c$ suppressed. They may be the dominant modes of the decays $\Lambda_b \to \Sigma_c^{(*)} X$, although they are phase space suppressed. We will calculate them in the heavy quark limit. The calculation involves in the HHCL by assuming the pion being soft. It also needs the three form factors of $\Lambda_b \to \Lambda_c$ and $\Sigma_b^{(*)} \to \Sigma_c^{(*)}$. In the large N_c limit, there is a light quark spin-flavor symmetry in the baryon sector. This leads to the fact that only one form factor is independent among the three [7]. It is argued that this result is true until to the order of $1/N_c^2$ [8].

This paper is organized as follows. In Sec. II, after a brief review of the HHCL, the invariant matrix elements of the decays are calculated in this framework with the help of the $1/N_c$ expansion. Sec. III describes necessary kinematics for the phase space integration and presents the numerical results. The discussion and summary are made in Sec. IV.

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II. DYNAMICS

The dynamics of the decays $\Lambda_b \to \Sigma_c^{(*)} \pi l \bar{\nu}$ is shown in Fig. 1, which includes both the strong and the weak interactions. The strong interaction is described by the HHCL in the case of the pion being soft. The chiral perturbation theory is the low energy effective theory of QCD for the light pseudoscalar mesons. These mesons are regarded as the Goldstone bosons due to the spontaneous chiral symmetry breaking $\mathrm{SU}(3)_L \times \mathrm{SU}(3)_R \to \mathrm{SU}(3)_V$ in QCD. The eight Goldstone boson fields, namely the pion, the kaon and the η-meson fields, can be represented in the nonlinear exponential form [9]

$$\Sigma = \exp(2i\tilde{\pi}/f) \ , \tag{1}$$

where

$$\tilde{\pi} = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{\frac{1}{2}}\pi^0 + \sqrt{\frac{1}{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\sqrt{\frac{1}{2}}\pi^0 + \sqrt{\frac{1}{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta \end{pmatrix} \ , \tag{2}$$

and f is the pion decay constant, $f = 93$ MeV. It is convenient to introduce

$$\xi \equiv \sqrt{\Sigma} = \exp(i\tilde{\pi}/f) \ . \tag{3}$$

Σ and ξ transform under the chiral symmetry as

$$\Sigma \to L\Sigma R^\dagger \ , \quad \xi \to L\xi U^\dagger = U\xi R^\dagger \ , \tag{4}$$

where $L(R) \in \mathrm{SU}(3)_{L(R)}$ and the special unitary matrix U is a complicated nonlinear function of L, R, and $\tilde{\pi}$. To the leading order of the derivative expansion, the effective Lagrangian for these Goldstone bosons is

$$\mathcal{L}_0 = \frac{1}{4} f^2 \mathrm{Tr}(\partial^\mu \Sigma^\dagger \partial_\mu \Sigma) \ . \tag{5}$$

Heavy baryons can be included in the chiral Lagrangian formalism. The ground state heavy baryons can be classified into two categories. One is the $\mathrm{SU}(3)_V$ sextet field S_μ^{ij} which

describes the heavy baryons whose light quarks have one unit of angular momentum. The resulting total angular momentum of these baryons is either 1/2 or 3/2. S_μ^{ij} is written as [10]

$$S_\mu^{ij}(v) = \sqrt{\frac{1}{3}}(\gamma_\mu + v_\mu)\gamma_5 \frac{1+\not{v}}{2} B^{ij} + \frac{1+\not{v}}{2} B_\mu^{*ij} \ , \tag{6}$$

where v is the 4-velocity of the heavy baryon. B^{ij} is the Dirac field and B_μ^{*ij} is its spin-3/2 counterpart. B^{ij} represents the following 3×3 matrix

$$B_{ij} = \begin{pmatrix} \Sigma_Q^{+1} & \frac{1}{\sqrt{2}}\Sigma_Q^0 & \frac{1}{\sqrt{2}}\Xi_Q^{\prime+\frac{1}{2}} \\ \frac{1}{\sqrt{2}}\Sigma_Q^0 & \Sigma_Q^{-1} & \frac{1}{\sqrt{2}}\Xi_Q^{\prime-\frac{1}{2}} \\ \frac{1}{\sqrt{2}}\Xi_Q^{\prime+\frac{1}{2}} & \frac{1}{\sqrt{2}}\Xi_Q^{\prime-\frac{1}{2}} & \Omega_Q \end{pmatrix} \ , \tag{7}$$

and B_μ^{*ij} has a similar form. The superscripts denote the charge and the isospin. Under the chiral symmetry, S_μ^{ij} transforms as

$$S_\mu^{ij} \to U_{i'}^i S_\mu^{i'j'} U_{j'}^j \ . \tag{8}$$

Under the heavy quark spin $\mathrm{SU}(2)_v$ symmetry,

$$S_\mu \to \exp^{\mathrm{i}\epsilon\cdot\mathbf{S}_v} S_\mu \ , \tag{9}$$

where $\mathbf{S}_v$ is the SU(2) generator and ϵ is an infinitesimal parameter. The other component in the classification is the $\mathrm{SU}(3)_V$ anti-triplet field T_i corresponding to the baryons whose light quarks have zero angular momentum. The resulting total angular momentum of the baryons is 1/2. It can be expressed as

$$T_i(v) = \frac{1+\not{v}}{2} B_i \ , \tag{10}$$

where the Dirac field B_i stands for

$$B_1 = \Xi_Q^{-\frac{1}{2}} \ , \quad B_2 = -\Xi_Q^{+\frac{1}{2}} \ , \quad B_3 = \Lambda_Q \ . \tag{11}$$

Under the chiral symmetry, it transforms as

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$$T_i \to T_j (U^\dagger)_i^j \ . \tag{12}$$

Under the $\mathrm{SU}(2)_v$, it transforms just like (9).

To the leading order of the derivative expansion, the effective Lagrangian of the strong interaction of S_μ^{ij} and T_i with the Goldstone bosons, which is invariant under the chiral symmetry and the heavy quark symmetry is [5]

$$\begin{aligned}\mathcal{L}_{int} = \sum_{\text{heavy quarks}} \{ &- \mathrm{i}\bar{S}_{ij}^\mu v\cdot\mathcal{D} S_\mu^{ij} + \mathrm{i}\bar{T}^i v\cdot\mathcal{D} T_i + \mathrm{i}g_2\epsilon_{\mu\nu\sigma\lambda}\bar{S}_{ik}^\mu v^\nu (A^\sigma)_j^i (S^\lambda)^{jk} \\ &+ g_3[\epsilon_{ijk}\bar{T}^i (A^\mu)_l^j S_\mu^{kl} + \epsilon^{ijk}\bar{S}_{kl}^\mu (A_\mu)_j^l T_i] + \Delta \bar{S}_{ij}^\mu S_\mu^{ij}\} \ ,\end{aligned} \tag{13}$$

where g_i's $(i = 1, 2, 3)$ denote the coupling constants and the covariant derivatives are

$$\begin{aligned}\mathcal{D}^\mu S_\nu^{ij} &= \partial^\mu S_\nu^{ij} + (V^\mu)_k^i S_\nu^{kj} + (V^\mu)_k^j S_\nu^{ik} \ , \\ \mathcal{D}^\mu T_i &= \partial^\mu T_i - T_j (V^\mu)_i^j \ ,\end{aligned} \tag{14}$$

and

$$\begin{aligned}\mathbf{V}^\mu &= \frac{1}{2}(\xi^\dagger\partial^\mu\xi + \xi\partial^\mu\xi^\dagger) \ , \\ \mathbf{A}^\mu &= \frac{1}{2}\mathrm{i}(\xi^\dagger\partial^\mu\xi - \xi\partial^\mu\xi^\dagger) \ .\end{aligned} \tag{15}$$

The mass difference between the sextet and the anti-triplet baryons is

$$\Delta = m_S - m_T \ . \tag{16}$$

We have neglected the mass differences between the Σ_Q and Σ_Q^* which is $1/m_Q$ suppressed.

The weak interaction of $b \to cl\nu$ is described by

$$\mathcal{L}_{\text{weak}} = \frac{4G_F}{\sqrt{2}} V_{cb} \sum_l (\bar{l}\gamma^\mu \frac{1-\gamma_5}{2}\nu_l)(\bar{c}\gamma_\mu \frac{1-\gamma_5}{2} b) \ , \tag{17}$$

and the hadronic current in the heavy quark limit can be re-expressed by the matter fields as

$$\begin{aligned}\bar{c}\gamma_\mu \frac{1-\gamma_5}{2} b \to C_{cb}\{&[-g_{\alpha\beta}\eta_1 (v\cdot v') + v_\alpha v'_\beta \eta_2 (v\cdot v')]\bar{S}_c^\alpha (v')\gamma_\mu \frac{1-\gamma_5}{2} S_b^\beta (v) \\ &+ \eta (v\cdot v')\bar{T}_c (v')\gamma_\mu \frac{1-\gamma_5}{2} T_b (v)\} \ ,\end{aligned} \tag{18}$$

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where η, η_1, and η_2 are the Isgur-Wise functions. And with the definition $\omega = v \cdot v'$, the perturbative QCD correction factor C_{cb} is

$$C_{cb}(v \cdot v') = \left[\frac{\alpha_s(m_b)}{\alpha_s(m_c)}\right]^{-\frac{6}{25}} \left[\frac{\alpha_s(m_c)}{\alpha_s(\mu)}\right]^{a_L(v\cdot v')} ,$$
$$a_L(w) = \frac{8}{27}\Big[\frac{w}{\sqrt{w^2-1}} \ln\left(w + \sqrt{w^2-1}\right) - 1\Big] . \tag{19}$$

In the large N_c limit, the Isgur-Wise functions have the following relations [7,8]

$$\eta(w) = \eta_1(w), \quad \eta_2(w) = \frac{\eta(w)}{w+1} . \tag{20}$$

In calculating Fig. 1, the spinor sum relations for the Dirac and Rarita-Schwinger types are used,

$$\sum_s u(v,s)\bar{u}(v,s) = \frac{1+\not{v}}{2} ,$$
$$\sum_s \mathcal{U}^\mu(v,s)\bar{\mathcal{U}}^\nu(v,s) = [-g^{\mu\nu} + v^\mu v^\nu + \frac{1}{3}(\gamma^\mu + v^\mu)(\gamma^\nu - v^\nu)] \cdot \frac{1+\not{v}}{2} , \tag{21}$$

where $u(v,s)$ is the Dirac spinor and $\mathcal{U}(v,s)$ is the Rarita-Schwinger spinor. We obtain the squared invariant matrix elements as

$$\frac{1}{2}\sum_{\text{spin}} |\mathcal{M}(\Lambda_b(v) \to \Sigma_c(v')\pi l\nu)|^2 = \frac{16G_F^2}{3f^2} g_3^2 |V_{cb}|^2 C_{cb}^2 \eta^2 \frac{p_l \cdot v'\ p_\nu \cdot v\ A}{(p_\pi \cdot v + \Delta)^2 (p_\pi \cdot v' + \Delta)^2} , \tag{22}$$

where

$$\begin{aligned} A = {} & 2(p_\pi \cdot v)^3 p_\pi \cdot v' + 2(p_\pi \cdot v)^3 \Delta + 4(p_\pi \cdot v)^2 (p_\pi \cdot v')^2 + 6(p_\pi \cdot v)^2 p_\pi \cdot v' \Delta \\ & + (p_\pi \cdot v)^2 \Delta^2 w + 3(p_\pi \cdot v)^2 \Delta^2 - (p_\pi \cdot v)^2 m_\pi^2 w - (p_\pi \cdot v)^2 m_\pi^2 \\ & + 2p_\pi \cdot v (p_\pi \cdot v')^3 + 6p_\pi \cdot v (p_\pi \cdot v')^2 \Delta - 2p_\pi \cdot v\ p_\pi \cdot v' \Delta^2 w + 2p_\pi \cdot v\ p_\pi \cdot v' \Delta^2 \\ & - 2p_\pi \cdot\ vp_\pi \cdot v' m_\pi^2 w - 2p_\pi \cdot v\ p_\pi \cdot v' m_\pi^2 - 4p_\pi \cdot v \Delta m_\pi^2 w - 4p_\pi \cdot v \Delta m_\pi^2 \\ & + 2(p_\pi \cdot v')^3 \Delta + (p_\pi \cdot v')^2 \Delta^2 w + 3(p_\pi \cdot v')^2 \Delta^2 - (p_\pi \cdot v')^2 m_\pi^2 w \\ & - (p_\pi \cdot v')^2 m_\pi^2 - 4p_\pi \cdot v' \Delta m_\pi^2 w - 4p_\pi \cdot v' \Delta m_\pi^2 - 4\Delta^2 m_\pi^2 w - 4\Delta^2 m_\pi^2) , \end{aligned} \tag{23}$$

and

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$$\frac{1}{2}\sum_{\rm spin}|\mathcal{M}(\Lambda_b(v)\to\Sigma_c^*(v')\pi l\nu)|^2=\frac{16G_F^2}{3f^2}g_3^2|V_{cb}|^2C_{cb}^2\eta^2\frac{B}{(p_\pi\cdot v+\Delta)^2(p_\pi\cdot v'+\Delta)^2}\ , \tag{24}$$

where

$$\begin{aligned}
B=&-p_l\cdot p_\nu\ (p_\pi\cdot v)^2\ (p_\pi\cdot v')^2w-p_l\cdot p_\nu\ (p_\pi\cdot v)^2\ (p_\pi\cdot v')^2+p_l\cdot p_\nu\ (p_\pi\cdot v)^2\ m_\pi^2w\\
&+p_l\cdot p_\nu\ (p_\pi\cdot v)^2\ m_\pi^2-2p_l\cdot p_\nu\ p_\pi\cdot v\ (p_\pi\cdot v')^2\ \Delta w-2p_l\cdot p_\nu\ p_\pi\cdot v\ (p_\pi\cdot v')^2\ \Delta\\
&+2p_l\cdot p_\nu\ p_\pi\cdot v\ \Delta m_\pi^2w+2p_l\cdot p_\nu\ p_\pi\cdot v\ \Delta m_\pi^2-p_l\cdot p_\nu\ (p_\pi\cdot v')^2\Delta^2w\\
&-p_l\cdot p_\nu\ (p_\pi\cdot v')^2\Delta^2+p_l\cdot p_\nu\ \Delta^2m_\pi^2w+p_l\cdot p_\nu\ \Delta^2m_\pi^2\\
&+4p_l\cdot v'\ p_\nu\cdot v\ (p_\pi\cdot v)^3\ p_\pi\cdot v'+4p_l\cdot v'\ p_\nu\cdot v\ (p_\pi\cdot v)^3\ \Delta\\
&-4p_l\cdot v'\ p_\nu\cdot v\ (p_\pi\cdot v)^2\ (p_\pi\cdot v')^2w+4p_l\cdot v'\ p_\nu\cdot v\ (p_\pi\cdot v)^2\ (p_\pi\cdot v')^2\\
&+12p_l\cdot v'\ p_\nu\cdot v\ (p_\pi\cdot v)^2\ p_\pi\cdot v'\Delta+2p_l\cdot v'\ p_\nu\cdot v\ (p_\pi\cdot v)^2\ \Delta^2w\\
&+6p_l\cdot v'\ p_\nu\cdot v\ (p_\pi\cdot v)^2\ \Delta^2+2p_l\cdot v'\ p_\nu\cdot v\ (p_\pi\cdot v)^2\ m_\pi^2w\\
&+2p_l\cdot v'\ p_\nu\cdot v\ (p_\pi\cdot v)^2\ m_\pi^2+4p_l\cdot v'\ p_\nu\cdot v\ p_\pi\cdot v\ (p_\pi\cdot v')^3\\
&-8p_l\cdot v'\ p_\nu\cdot v\ p_\pi\cdot v\ (p_\pi\cdot v')^2\ \Delta w+4p_l\cdot v'\ p_\nu\cdot v\ p_\pi\cdot v\ (p_\pi\cdot v')^2\ \Delta\\
&-4p_l\cdot v'\ p_\nu\cdot v\ p_\pi\cdot v\ p_\pi\cdot v'\ \Delta^2w+4p_l\cdot v'\ p_\nu\cdot v\ p_\pi\cdot v\ p_\pi\cdot v'\ \Delta^2\\
&-4p_l\cdot v'\ p_\nu\cdot v\ p_\pi\cdot v\ p_\pi\cdot v'\ m_\pi^2w-4p_l\cdot v'\ p_\nu\cdot v\ p_\pi\cdot v\ p_\pi\cdot v'\ m_\pi^2\\
&+4p_l\cdot v'\ p_\nu\cdot v\ (p_\pi\cdot v')^3\Delta-2p_l\cdot v'\ p_\nu\cdot v\ (p_\pi\cdot v')^2\Delta^2w\\
&+2p_l\cdot v'\ p_\nu\cdot v\ (p_\pi\cdot v')^2\Delta^2-2p_l\cdot v'\ p_\nu\cdot v\ (p_\pi\cdot v')^2m_\pi^2w-2p_l\cdot v'\ p_\nu\cdot v\ (p_\pi\cdot v')^2m_\pi^2\\
&-8p_l\cdot v'\ p_\nu\cdot v\ p_\pi\cdot v'\ \Delta\ m_\pi^2w-8p_l\cdot v'\ p_\nu\cdot v\ p_\pi\cdot v'\ \Delta\ m_\pi^2-4p_l\cdot v'\ p_\nu\cdot v\ \Delta^2\ m_\pi^2w\\
&-4p_l\cdot v'\ p_\nu\cdot v\ \Delta^2\ m_\pi^2\ .
\end{aligned}\tag{25}$$

In Eqs. (22−25), p_π, p_l and p_ν are the momentums of the pion, lepton and neutrino, respectively.

III. KINEMATICS AND RESULTS

The kinematics of the four-body decay has been described in Ref. [11]. It is shown in Fig. 2. Under the definition of

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$$s_H \equiv (p_{\Sigma_c^{(*)}} + p_\pi)^2 \ , \quad s_L \equiv (p_l + p_\nu)^2 \ , \tag{26}$$

the total decay rate is given by

$$\begin{aligned} d\Gamma &= \frac{1}{2m_{\Lambda_b}} \frac{d^3p_{\Sigma_c^{(*)}}}{(2\pi)^3 2E_{\Sigma_c^{(*)}}} \frac{d^3p_l}{(2\pi)^3 2E_l} \frac{d^3p_\nu}{(2\pi)^3 2E_\nu} \frac{d^3p_\pi}{(2\pi)^3 2E_\pi} \\ &\quad \times (2\pi)^4 \delta^4(p_{\Lambda_b} - p_{\Sigma_c^{(*)}} - p_l - p_\nu - p_\pi) \left(\frac{1}{2} \sum_{\text{spin}} |\mathcal{M}|^2 \right) \\ &= \frac{1}{4(4\pi)^6 m_{\Lambda_b}^3} X\beta ds_H ds_L d\cos\theta d\cos\theta_l d\phi \left(\frac{1}{2} \sum_{\text{spin}} |\mathcal{M}|^2 \right) \ , \end{aligned} \tag{27}$$

where

$$\begin{aligned} X &= \frac{1}{2}(m_{\Lambda_b}^4 + s_H^2 + s_L^2 - 2m_{\Lambda_b}^2 s_H - 2s_H s_L - 2m_{\Lambda_b}^2 s_L)^{\frac{1}{2}} = m_{\Lambda_b} \sqrt{(E_{\Sigma_c^{(*)}} + E_\pi)^2 - s_H} \ , \\ \beta &= \frac{1}{s_H}[(s_H + m_\pi^2 - m_{\Sigma_c^{(*)}}^2)^2 - 4m_\pi^2 s_H]^{\frac{1}{2}} \ , \end{aligned} \tag{28}$$

and $E_{\Sigma_c^{(*)}}$, E_π are measured in Λ_b rest frame.

To get the decay rate distributions, the following values are used,

$$\begin{aligned} &m_{\Lambda_b} = 5.64 \text{ GeV}, \quad m_{\Sigma_c^{(*)}} = 2.453 \text{ GeV}, \quad f = 0.093 \text{ GeV}, \\ &V_{cb} = 0.041, \quad \Lambda_{\text{QCD}} = 0.2 \text{ GeV}, \quad \Delta = 0.2 \text{ GeV} \ . \end{aligned} \tag{29}$$

The value of g_3 is 0.99 [12]. We adopt two forms for $\eta(w)$. One is a linear fitting of the result from QCD sum rule $\eta(w) = 1 - 0.55(w-1)$ [13]; and the other is an exponential function from large N_c QCD $\eta(w) = 0.99\exp[-1.3(w-1)]$ [14].

The numerical results of $\Lambda_b \to \Sigma_c \pi l \bar{\nu}$ and $\Lambda_b \to \Sigma_c^* \pi l \bar{\nu}$ are given in Figs. 3 and 4, respectively. The decay rate distributions on $\omega = v \cdot v'$, pion energy and lepton energy are shown. The results of the two kinds of Isgur-Wise function are compared.

The total decay rates are obtained as

$$\begin{aligned} &\Gamma(\Lambda_b \to \Sigma_c \pi l \bar{\nu}) = 6.45 \times 10^{-15} \ (5.20 \times 10^{-15}) \text{ GeV}, \\ &\Gamma(\Lambda_b \to \Sigma_c^* \pi l \bar{\nu}) = 4.27 \times 10^{-15} \ (3.43 \times 10^{-15}) \text{ GeV}, \end{aligned} \tag{30}$$

and hence the branching ratios are

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$$\mathcal{B}(\Lambda_b \to \Sigma_c \pi l \bar{\nu}) = 1.21\ (0.971)\ \%\ ,$$
$$\mathcal{B}(\Lambda_b \to \Sigma_c^* \pi l \bar{\nu}) = 0.798\ (0.640)\ \%\ , \tag{31}$$

for the linear (exponential) $\eta(w)$. Compared with the result of $\Lambda_b \to \Lambda_c l\nu$ [15], $\Gamma(\Lambda_b \to \Sigma_c \pi l\nu)/\Gamma(\Lambda_b \to \Lambda_c l\nu) \simeq 10.5\ (11.5)\ \%$, and $\Gamma(\Lambda_b \to \Sigma_c^* \pi l\nu)/\Gamma(\Lambda_b \to \Lambda_c l\nu) \simeq 6.94.\ (7.59)\ \%$. They are less dependent on the Isgur-Wise function because of the partial cancellation of it in the ratios.

IV. SUMMARY AND DISCUSSION

We have calculated the decays $\Lambda_b \to \Sigma_c^{(*)} \pi l \bar{\nu}$ in the heavy quark, chiral and large N_c limits. The differential decay spectra are presented. The branching ratios are obtained as $\mathcal{B}(\Lambda_b \to \Sigma_c \pi l \bar{\nu}) = 1.21\ (0.971)\ \%$, and $\mathcal{B}(\Lambda_b \to \Sigma_c^* \pi l \bar{\nu}) = 0.798\ (0.640)\ \%$ with $\eta(w)$ from QCD sum rule (large N_c QCD).

Let us discuss the accuracy of these approximations. First, consider the finite heavy quark mass correction. We have taken $m_Q \to \infty$. Generally the size of the $1/m_Q$ correction can be around $\Lambda_{\rm QCD}/m_c \simeq 20\%$. The detailed consideration indicates that the correction maybe smaller. In the calculation, there are two places where the heavy quark limit is applied. One is in the weak transition where the quark current was replaced by the baryonic matrix elements. For the $\Lambda_b \to \Lambda_c$ transition, the $1/m_Q$ correction was calculated to be 11% [15]. However, there is no estimation for the $1/m_Q$ correction to the $\Sigma_b^{(*)} \to \Sigma_c^{(*)}$ transition. It is experiential to guess that the correction in this case is also at the 10% level. The other place where the heavy quark limit was used is in the description of the strong interaction, namely in the HHCL. The difference between the coupling g_3 for the bottom quark and that for the charm quark, and the difference between the masses of Σ_Q and Σ_Q^*, have been neglected. Theoretically these differences are both $1/m_Q$ and $1/N_c^2$ suppressed [4,16]. Note that practically $m_{\Sigma_c^*} - m_{\Sigma_c} \simeq 60$ MeV [1] which is smaller than the pion mass. Hence the correction in the second place is very small $\sim \frac{\Lambda_{\rm QCD}}{N_c^2 m_c} \approx 2\%$. From the above discussion, we

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expect that the $1/m_Q$ correction to our results is less than 20%, and is probably around 10%.

Second, the validity of the soft pion limit should be addressed. The calculation is valid in the phase space region where the pion energy is smaller than the chiral symmetry breaking scale which is argued to be at $4\pi f \simeq 1.2$ GeV [9]. Therefore for a large part of phase space, the calculation is expected to be reliable. However, the subleading terms in the derivative expansion in the effective Lagrangian have been omitted. And because of the unknown coefficients of the subleading terms, it is still not possible to include them in the calculation. The correction due to these terms is at the order of $v \cdot p_\pi/4\pi f$ or $v' \cdot p_\pi/4\pi f$ which is less than 30% if the pion is really soft, say $E_\pi \leq 0.5$ GeV. For the total decay rate, the results we have obtained maybe subject to larger correction.

Finally, the $1/N_c$ expansion used in the calculation needs to be explicated. The calculation needs the information of the three Isgur-Wise functions of the transitions $\Lambda_b \to \Lambda_c$ and $\Sigma_b^{(*)} \to \Sigma_c^{(*)}$. In the large N_c limit, the three form factors are related to each other [7]. These relations are argued to be valid to the order of $1/N_c^2$ [8] due to the light quark spin-flavor symmetry in the baryon sector in the large N_c limit. The argument goes like that as in Ref. [4]. It is the relation among the form factors, not the knowledge about the form factor itself, that we are considering. Therefore the application of these relations causes $1/N_c^2 \simeq 10\%$ uncertainty.

Up to the above approximations, the calculation is in principle model-independent. To get the final numerical results, the model result for the $\Lambda_b \to \Lambda_c$ Isgur-Wise function has to be used. This step brings one more source of uncertainty. According to the model results we adopted, the uncertainty is at 20% level.

The decays $\Lambda_b \to \Sigma_c^{(*)} \pi l \bar{\nu}$ are indeed experimentally interesting according to our results. The estimated branching ratios of the decays are not small. The experiment has been studying the channel $\Lambda_b \to \Lambda_c l \bar{\nu}$ [17]. A lot of data on Λ_b can be expected to emerge in the near future from CDF, D0, HERA-B, and LHC-B and so on. With high statistics of the data, the decay channels $\Lambda_b \to \Sigma_c^{(*)} \pi l \bar{\nu}$ can be discovered.

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In the near future, more reliable theoretical results for the baryonic Isgur-Wise functions can be obtained, even for that appear in the subleading order of the heavy quark expansion, from lattice HQET or the other model-independent methods. That will give more accurate results of the decay distributions and decay rates. The uncertainties due to the heavy quark limit, large N_c limit and the model-dependence can be reduced to 10% or smaller. In this case, the precise experimental measurement for $\Lambda_b \to \Sigma_c^{(*)} \pi l \bar{\nu}$ will give more information about the validity range of the HHCL.

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FIGURE CAPTIONS

Fig. 1 Tree level diagrams for $\Lambda_b \to \Sigma_c^{(*)} \pi l \nu$. Solid square(circle) indicates the weak(strong) interaction vertex.

Fig. 2 The kinematics of $\Lambda_b \to \Sigma_c^{(*)} \pi l \nu$. Two planes represent the final state hadron and lepton c.m. frame, respectively. The dashed line is the flight line of s_H and s_L.

Fig. 3 Decay rate distributions for $\Lambda_b \to \Sigma_c \pi l \nu$ over (a) $w = v \cdot v'$, (b) pion energy, and (c) lepton energy. Solid lines are the results of linear $\eta(w)$ (QCD sum rule), while the dotted lines are the ones of exponential $\eta(w)$ (large N_c). All the energy scales are in GeV, and the distributions are plotted in units of 10^{-14} or 10^{-15} GeV.

Fig. 4 Decay rate distributions for $\Lambda_b \to \Sigma_c^* \pi l \nu$ over (a) $w = v \cdot v'$, (b) pion energy, and (c) lepton energy. Solid lines are the results of linear $\eta(w)$ (QCD sum rule), while the dotted lines are the ones of exponential $\eta(w)$ (large N_c). All the energy scales are in GeV, and the distributions are plotted in units of 10^{-14} or 10^{-15} GeV.

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FIGURES

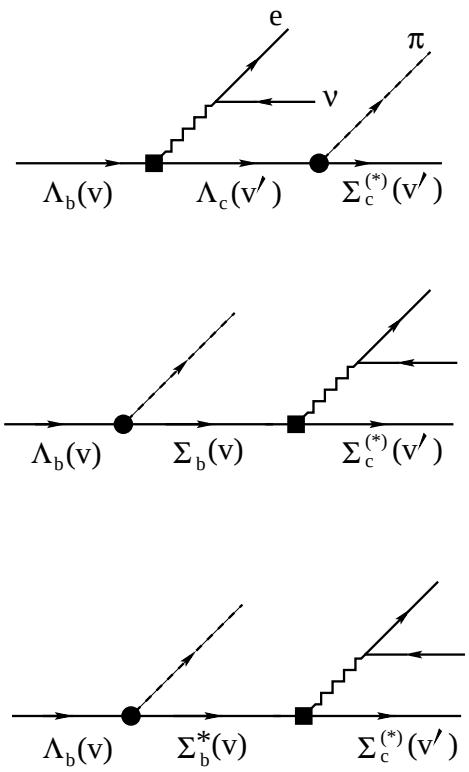
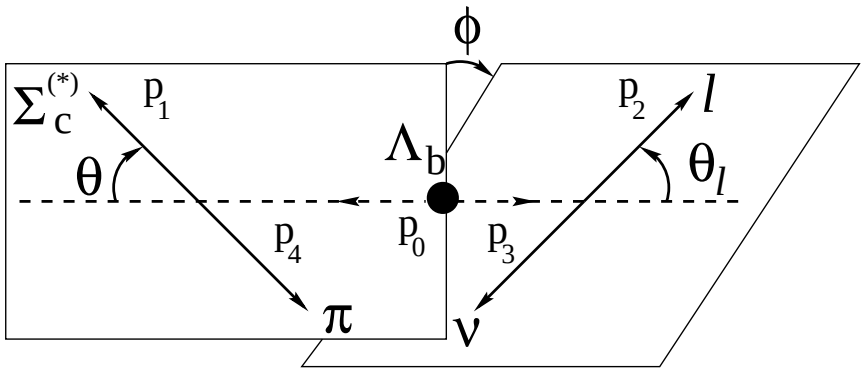


FIG. 1.



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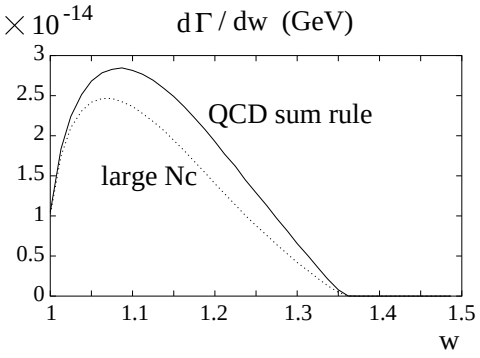

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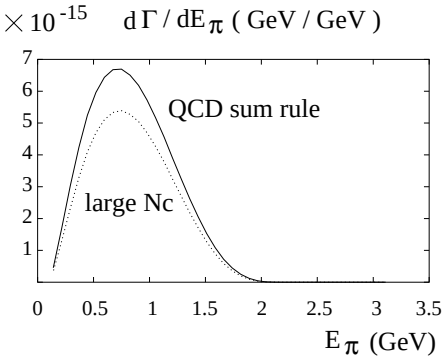

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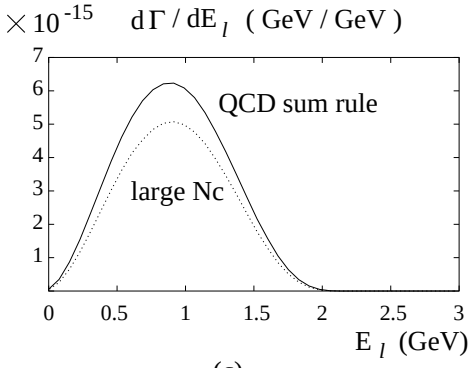

(c)

FIG. 3.

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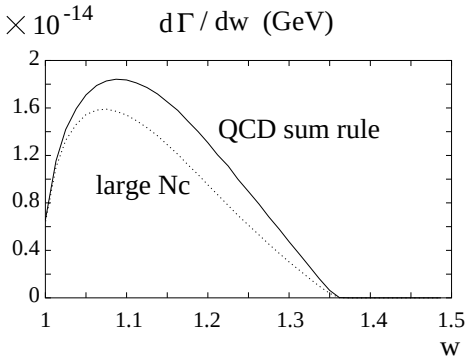

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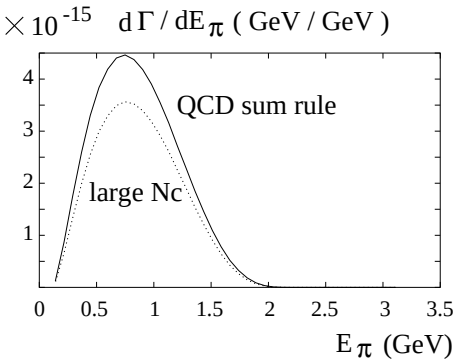

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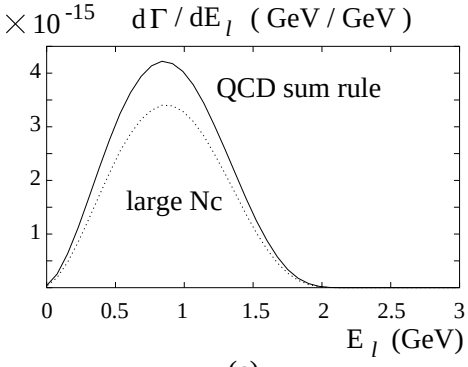

(c)

FIG. 4.

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